Unification of Inflation and Dark Energy from Spontaneous Breaking of Scale Invariance

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Abstract

We propose a new class of gravity-matter models defined in terms of two independent non-Riemannian volume forms (alternative generally covariant integration measure densities) on the spacetime manifold. For the matter we choose appropriate scalar field potentials of exponential form so that the full gravity-matter system is invariant under global scale symmetry. Solution of the pertinent equations of motion produce two dimensionful integration constants which spontaneously break scale invariance. In the resulting effective Einstein-frame gravity-matter system we obtain an effective potential for the scalar matter field which has an interesting cosmological application, namely, it allows for a unified description of both an early universe inflation and present day dark energy.

1 Introduction

A component of the energy-momentum tensor of matter which is proportional to the spacetime metric tensor, with the proportionality constant being indeed exactly or approximately a spacetime constant, has been widely discussed and its consequences understood, but the possible origin of such terms remains a subject of hot discussion.

One can transfer such energy-momentum tensor component to the left hand side of Einstein's equations and then it can be considered as belonging to the gravity part. This was the way indeed how Einstein introduced such contribution and named it the "cosmological constant term".

More recently it has been invoked as a fundamental component of the energy density of both the early universe and of the present universe. Nowadays we call such component "vacuum energy density". The vacuum energy density has been used as the source of a possible inflationary phase of the early universe (the pioneering papers on the subject are [1]; for a non-technical review and a good collection of further references on different aspects of inflation see Ref.[2]; for a more technical review see Ref.[3]). Inflation provides an attractive scenario for solving some of the fundamental puzzles of the standard Big Bang model, like the horizon and the flatness problems (third ref.[1]) as well as providing a framework for sensible calculations of primordial density perturbations (for a review, see the book [4]).

Also, with the discovery of the accelerating expansion of the present universe (for reviews of this subject, see for example [5, 6]) it appears plausible that a small vacuum energy density, usually referred in this case as "dark energy", is also present even today. Because of this discovery the cosmological constant problem (CCP) has evolved from the "Old Cosmological Constant Problem" [7], where physicists were concerned with explaining why the observed vacuum energy density of the universe nowadays is vanishing, to a different type of CCP – the "New Cosmological Constant Problem" [8]. Namely, the problem now is to explain why the vacuum energy density of the current universe is very small rather than being zero.

These two vacuum energy densities, the one of inflation and the other of the universe nowadays, have however a totally different scale. One then wonders how cosmological evolution may naturally interpolate between such two apparently quite distinctive physical situations.

The possibility of continuously connecting an inflationary phase to a slowly accelerating universe through the evolution of a single scalar field – the *quintessential inflation scenario* – has been first studied in Ref.[9]. Also, carefully constructed F(R) models can yield both an early time inflationary epoch and a late time de Sitter phase with vastly different values of effective vacuum energies [10]. For a recent proposal of a quintessential inflation mechanism based on "variable gravity" model [11] and for extensive list of references to earlier work on quintessential inflation, see Ref.[12].

In the present letter we propose a new theoretical framework where the quintessential inflation scenario is explicitly realized in a natural way.

The main idea of our current approach comes from Refs.[13, 14, 15] (for recent developments, see Refs.[16]), where some of us have proposed a new class of gravity-matter theories based on the idea that the action integral may contain a new metric-independent integration measure density, *i.e.*, an alternative non-Riemannian volume form on the spacetime manifold defined in terms of an auxiliary antisymmetric gauge field of maximal rank. The latter formalism yields various new interesting results in all types of known generally coordinate-invariant theories:

- (i) $D = 4$-dimensional models of gravity and matter fields containing the new measure of integration appear to be promising candidates for resolution of the dark energy and dark matter problems, the fifth force problem, and a natural mechanism for spontaneous breakdown of global scale symmetry [13]-[16].
- (ii) Study of reparametrization invariant theories of extended objects (strings and branes) based on employing of a modified non-Riemannian world-sheet/world-volume integration measure [17] leads to dynamically induced variable string/brane tension and to string models of non-abelian confinement.
- (iii) Study in Refs.[18] of modified supergravity models with an alternative non-Riemannian volume form on the spacetime manifold produces some outstanding new features: (a) This new formalism applied to minimal $N = 1$ supergravity naturally triggers the appearance of a dynamically generated cosmological constant as an arbitrary integration constant, which signifies a new explicit mechanism of spontaneous (dynamical) breaking of supersymmetry; (b) Applying the same formalism to anti-de Sitter supergravity allows us to appropriately choose the above mentioned arbitrary integration constant so as to obtain simultaneously a very small effective observable cosmological constant as well as a very large physical gravitino mass.

We now extend the above formalism employing two (instead of only one) modified non-Riemannian volume-forms on the underlying spacetime to construct new type of gravity-matter models producing interesting cosmological implications relating inflationary and slowly accelerating phases of the universe.

2 Gravity-Matter Models With Two Independent Non-Riemannian Volume-Forms

We shall consider the following non-standard gravity-matter system with an action of the general form (for simplicity we will use units where the Newton constant is taken as $G_N = 1/16\pi$):

$$S = \int d^4 x\, \Phi_1(A) \Big[R + L^{(1)} \Big] + \int d^4 x\, \Phi_2(B) \Big[L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} \Big] \; , \tag{1}$$

with the following notations:

- $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms, *i.e.*, generally covariant integration measure densities on the underlying spacetime manifold:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \; , \tag{2}$$

  defined in terms of field-strengths of two auxiliary 3-index antisymmetric tensor gauge fields. $\Phi_{1,2}$ take over the role of the standard Riemannian integration measure density $\sqrt{-g} \equiv \sqrt{-\det \|g_{\mu\nu}\|}$ in terms of the spacetime metric $g_{\mu\nu}$.

- $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ are the scalar curvature and the Ricci tensor in the first-order (Palatini) formalism, where the affine connection $\Gamma^{\mu}_{\nu\lambda}$ is *a priori* independent of the metric $g_{\mu\nu}$.
- $L^{(1,2)}$ denote two different Lagrangians with matter fields, to be specified below.
- $\Phi(H)$ indicate the dual field strength of a third auxiliary 3-index antisymmetric tensor gauge field:

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}; , \tag{3}$$

  whose presence is crucial for non-triviality of the model.

For the matter Lagrangians we take the scalar field ones:

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) \quad , \quad L^{(2)} = U(\varphi) \;\; \text{(no kinetic term)} \; . \tag{4}$$

We now observe that the original action (1) is invariant under global scale transformations:

$$\begin{gathered} g_{\mu\nu} \to \lambda g_{\mu\nu} \;\; , \;\; \varphi \to \varphi - \frac{1}{\alpha}\ln\lambda \;\; , \\ A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \;\; , \;\; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \;\; , \;\; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} \; , \end{gathered} \tag{5}$$

where α is a dimensionful positive parameter, provided we choose the scalar field potentials in (4) in the form (similar to the choice [13]):

$$V(\varphi) = f_1 \exp\{\alpha\varphi\} \quad , \quad U(\varphi) = f_2 \exp\{2\alpha\varphi\} \; . \tag{6}$$

Variation of (1) w.r.t. $\Gamma^{\mu}_{\nu\lambda}$ gives (following the derivation in [13]):

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda}\right) \; , \tag{7}$$

where $\bar{g}_{\mu\nu}$ is the Weyl-rescaled metric:

$$\bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu} \;\; , \;\; \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \; . \tag{8}$$

Variation of the action (1) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu \Big[R + L^{(1)}\Big] = 0 \quad , \quad \partial_\mu \Big[L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}}\Big] = 0 \quad , \quad \partial_\mu \Big(\frac{\Phi_2(B)}{\sqrt{-g}}\Big) = 0 \; , \tag{9}$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} = \chi_2 = \text{const} \;\; , \;\; R + L^{(1)} = -M_1 = \text{const} \;\; , \;\; L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const} \; . \tag{10}$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimenionless integration constants. The appearance of M_1, M_2 signifies *dynamical spontaneous breakdown* of scale invariance under (5) due to the scale non-invariant solutions (second and third ones) in (10).

Varying (1) w.r.t. $g_{\mu\nu}$ and using relations (10) we have:

$$\chi_1 \Big[R_{\mu\nu} + \frac{\partial}{\partial g^{\mu\nu}} L^{(1)}\Big] - \frac{1}{2}\chi_2 \Big[T^{(2)}_{\mu\nu} + g_{\mu\nu} M_2\Big] = 0 \; , \tag{11}$$

where χ_1 and χ_2 are defined in (8) and first relation (10), and $T^{(2)}_{\mu\nu}$ is the energy-momentum tensor of the second matter Lagrangian with the standard definitions:

$$T^{(1,2)}_{\mu\nu} = g_{\mu\nu} L^{(1,2)} - 2\frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)} \; . \tag{12}$$

Using second relation (10) and (12), Eqs.(11) can be put in the form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}\Big[T^{(1)}_{\mu\nu} + g_{\mu\nu}M_1 + \frac{\chi_2}{\chi_1}\Big(T^{(2)}_{\mu\nu} + g_{\mu\nu}M_2\Big)\Big] . \tag{13}$$

Taking the trace of Eqs.(13) and using again second relation (10) we solve for the scale factor χ_1:

$$\chi_1 = 2\chi_2\frac{U(\varphi)+M_2}{V(\varphi)-M_1} . \tag{14}$$

Now, taking into account (8) and (14) we can bring Eqs.(13) into the standard form of Einstein equations for the metric $\bar{g}_{\mu\nu}$ (8) , *i.e.*, the Einstein frame equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\rm eff}_{\mu\nu} \tag{15}$$

with energy-momentum tensor corresponding (according to (12)) to the following effective scalar field Lagrangian:

$$L_{\rm eff} = -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U_{\rm eff}(\varphi) , \tag{16}$$

where the effective scalar field potential reads:

$$U_{\rm eff}(\varphi) = \frac{\big(V(\varphi)-M_1\big)^2}{4\chi_2\big(U(\varphi)+M_2\big)} = \frac{\left(f_1 e^{\alpha\varphi}-M_1\right)^2}{4\chi_2\ \left(f_2 e^{2\alpha\varphi}+M_2\right)} . \tag{17}$$

3 Implications for Cosmology

The effective scalar potential (17) possesses the following remarkable property. For large negative and large positive values of φ $U_{\rm eff}(\varphi)$ exponentially fast approaches two flat regions with smooth transition between them:

$$U_{\rm eff}(\varphi) \to \frac{M_1^2}{4\chi_2 M_2} \quad \text{for} \ \ \varphi \to -\infty ,$$
$$U_{\rm eff}(\varphi) \to \frac{f_1^2}{4\chi_2 f_2} \quad \text{for} \ \ \varphi \to +\infty . \tag{18}$$

The shape of $U_{\rm eff}(\varphi)$ depicted on Fig.1.

In the original gravity-matter models with only one modified non-Riemannian volume form [13] one obtains upon spontaneous breakdown of scale symmetry only one flat region of the effective scalar potential, so that this simple model does not meet the requirement for unification of inflation and dark energy.

Let us point out that in the context of the original modified-measure gravity-matter theories (with only one non-Riemannian integration measure density) it is possible to obtain two flat regions by means of adding an ϵR^2 term [15]. However, the latter procedure makes the scalar field non-canonical and non-linear scalar field kinetic terms do appear which substantially complicates the theory and its particle content interpretation.

In the present case we derived an effective scalar potential $U_{\rm eff}(\varphi)$ (17) with *two flat regions* while the kinetic term of the scalar field remained canonical. In the course of the derivation we obtained three integration constants χ_2, M_1, M_2 (10), two of them (M_1, M_2) triggering spontaneous breakdown of the original scale symmetry (5). These integration constants can be appropriately adjusted so as to get the shape of the effective scalar potential as depicted on Fig.1.

The cosmological picture suggested by Fig.1 is evident. The universe starts from a large negative value of φ, then slow rolls the first flat region to the left whose height $U_{\rm eff}(\varphi) \simeq \frac{M_1^2}{4\chi_2 M_2}$ upon appropriate choce of M_1, M_2 can be made very large corresponding to the vacuum energy density in the inflationary phase. After that there is an abrupt fall to $U_{\rm eff} = 0$ where particle creation is obtained from rapidly varying $\varphi(t)$. The scalar field comes down with very high kinetic energy in the region of $U_{\rm eff} \simeq 0$, certainly higher than

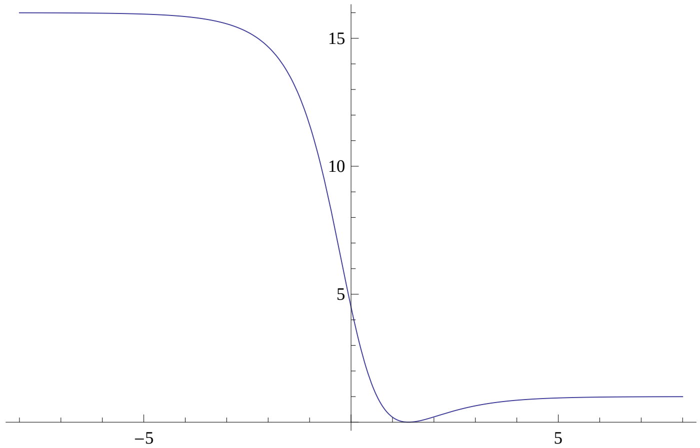

Figure 1: Shape of the effective scalar potential $U_{\rm eff}(\varphi)$ (17).

the value of $U_{\rm eff}$ in the second flat region to the right $U_{\rm eff}(\varphi) \simeq \frac{f_1^2}{4\chi_2 f_2}$, which upon appropriate choice of the scales of f_1, f_2 (see below) can be made to correspond to the correct value of the current vacuum energy density. So $\varphi(t)$ "climbs" the latter low barrier and continues to evolve in the $\varphi \to +\infty$ direction. Thus, on the second flat region we have a slow rolling scalar which produces approximately the dark energy equation of state ($\rho \simeq -p$, with very small $\rho \simeq \frac{f_1^2}{4\chi_2 f_2}$) explaining the present day dark energy phase.

Indeed, taking the integration constant $\chi_2 \sim 1$, and choosing the scales of the scalar potential (17) coupling constants $f_1 \sim M_{EW}^4$ and $f_2 \sim M_{Pl}^4$, where M_{EW}, M_{Pl} are the electroweak and Plank scales, respectively, we are then naturally led to a very small vacuum energy density of the order M_{EW}^8/M_{Pl}^4. The fact that the ratio M_{EW}^8/M_{Pl}^4 gives the right order of magnitude for the present vacuum energy density was already recognized in Ref.[19].

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